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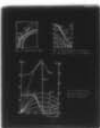
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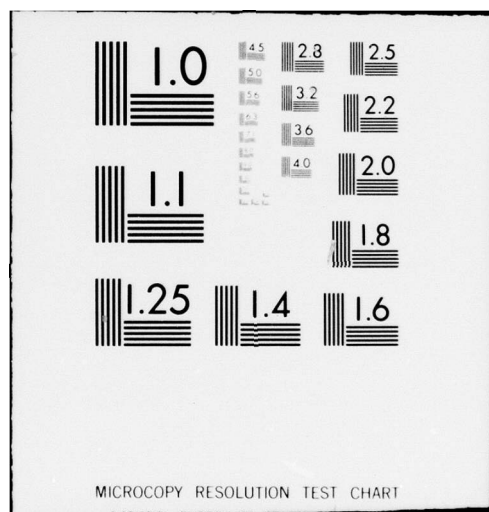
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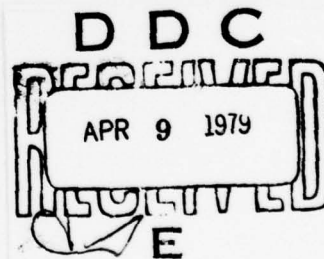
FOREIGN TECHNOLOGY DIVISION



CALCULATION OF A SUPERSONIC FLOW AROUND A COOLABLE  
SPHERE USING THE COMPLETE NAVIER-STOKES EQUATIONS

by

B. M. Pavloa



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## EDITED TRANSLATION

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# U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<b>А а</b>	A, a	Р р	<b>Р р</b>	R, r
Б б	<b>Б б</b>	B, b	С с	<b>С с</b>	S, s
В в	<b>В в</b>	V, v	Т т	<b>Т т</b>	T, t
Г г	<b>Г г</b>	G, g	У у	<b>У у</b>	U, u
Д д	<b>Д д</b>	D, d	Ф ф	<b>Ф ф</b>	F, f
Е е	<b>Е е</b>	Ye, ye; E, e*	Х х	<b>Х х</b>	Kh, kh
Ж ж	<b>Ж ж</b>	Zh, zh	Ц ц	<b>Ц ц</b>	Ts, ts
З з	<b>З з</b>	Z, z	Ч ч	<b>Ч ч</b>	Ch, ch
И и	<b>И и</b>	I, i	Ш ш	<b>Ш ш</b>	Sh, sh
Й й	<b>Й й</b>	Y, y	Щ щ	<b>Щ щ</b>	Shch, shch
К к	<b>К к</b>	K, k	Ъ ъ	<b>Ъ ъ</b>	"
Л л	<b>Л л</b>	L, l	Ы ы	<b>Ы ы</b>	Y, y
М м	<b>М м</b>	M, m	Ь ь	<b>Ь ь</b>	'
Н н	<b>Н н</b>	N, n	Э э	<b>Э э</b>	E, e
О о	<b>О о</b>	O, o	Ю ю	<b>Ю ю</b>	Yu, yu
П п	<b>П п</b>	P, p	Я я	<b>Я я</b>	Ya, ya

\*ye initially, after vowels, and after Ъ, Ь; e elsewhere.  
When written as ё in Russian, transliterate as yë or ë.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	sinh <sup>-1</sup>
cos	cos	ch	cosh	arc ch	cosh <sup>-1</sup>
tg	tan	th	tanh	arc th	tanh <sup>-1</sup>
ctg	cot	cth	coth	arc cth	coth <sup>-1</sup>
sec	sec	sch	sech	arc sch	sech <sup>-1</sup>
cosec	csc	csch	csch	arc csch	csch <sup>-1</sup>

Russian      English

rot      curl  
lg      log

CALCULATION OF A SUPERSONIC FLOW  
AROUND A COOLABLE SPHERE USING THE  
COMPLETE NAVIER-STOKES EQUATIONS

B. M. Pavlov

This work [1] presents a statement and a numerical solution for a problem of a stationary flow around blunt solids by a supersonic flow of a low-density gas. The method is based on a finite-difference approximation of the complete nonstationary Navier-Stokes equations for a compressible gas and on finding a stationary solution as a result of the determination. Works [1]-[3] present the initial calculation results and examine certain methodological problems which characterize the numerical algorithm. Work [4] contains the calculation results for a flow past a heat-insulated sphere. This work discusses a pattern of flow around a coolable sphere under the conditions of small Reynolds numbers.

**Designations:**  $r, \theta$  - spherical polar system of coordinates (Fig. 1);  $r_w$  - sphere radius;  $W$  - flow velocity vector;  $u, v$  - its components with respect to  $r$  and  $\theta$ ;  $\rho, h, T, p$  - density, enthalpy, temperature, and pressure;  $\lambda, \mu$  - coefficients of heat conductivity and shear viscosity;  $\gamma$  - ratio of specific heats;  $q$  - heat flow to the wall;  $\tau$  - friction tension;  $F$  - force of resistance;  $C_D$  - coefficient of total resistance;  $C_f$  - friction coefficient;  $l$  - mean free path of molecules;  $Re, M_\infty, Kn, Pr$  - numbers of Reynolds, Mach, Knudsen, and Prandl;  $d, I$  - thickness and intensity of a shock wave;  $p_0$  - stagnation pressure behind a normal shock;



$r_*$  - sonic line; subscripts:  $\infty$  - in an incident flow,  $w$  - on the sphere surface,  $s$  - behind a normal inviscid shock,  $0$  - flow stagnated adiabatically and isentropically,  $\Pi C$  - in a boundary layer,  $CM$  - in a free-molecular flow; superscripts:  $0$  - in the leading critical point,  $*$  - along the boundary ray  $\theta^*$ .

1. An examination is made of a viscous gas flow, which is uniform and supersonic at infinity, around a sphere. The problem concerning the calculation of the flow field is solved under the following assumptions: 1) the gas flowing past the sphere is thermally and calorically ideal, 2) the coefficients  $\lambda$  and  $\mu$  are power functions of enthalpy ( $\sim h^\omega$ ,  $0.5 \leq \omega \leq 1$ ), 3) the flow has a laminar nature at small Reynolds numbers, and 4) under these conditions of flow the Navier-Stokes equations for the compressible gas are valid in the entire area outside the sphere.

Conditions 1) and 2) lead to constant values of the numbers  $\gamma$  and  $Pr$ . Dimensionless variables are used: linear dimensions pertain to  $r_w$ , velocities - to  $W_\infty$ , density - to  $\rho_\infty$ , enthalpy - to  $W_\infty^2$ , pressure and tension of friction - to  $\rho_\infty W_\infty^2$ , heat flow - to  $\rho_\infty W_\infty^3$ , and force of resistance - to  $1/2 \pi r_w^2 \rho_\infty W_\infty^2$ .

Work [1] shows the initial system of equations in a dimensionless form relative to the unknowns  $\rho$ ,  $u$ ,  $v$ ,  $h$ ,  $p$ , and  $\mu$  in the system of coordinates  $r$ ,  $\theta$ . Since with a laminar nature the flow has a symmetry relative to the  $\theta=0$  axis, it is sufficient to seek a solution for the problem in the region  $r \geq 1$ ,  $0 \leq \theta \leq \pi$ . According to the systematic investigations, which are described briefly in [1] and [3], numerical integration can be accomplished in the end region (Fig. 1)  $1 \leq r \leq R_2$ ,  $0 \leq \theta \leq \theta^*$ , where  $R_2 = R_2(\theta)$  - given equation for the surface of rotation (in this case, of an ellipsoid whose left focal point is congruent with the center of the sphere),  $\theta^*$  - position of the boundary ray ( $0 < \theta^* \leq \pi$ ).

Let us formulate the boundary conditions of the problem. The conditions of the uniformity of the flow at an infinite distance from the sphere are removed to the surface  $R_2(\theta)$ ,  $0 \leq \theta \leq \theta^*$ . On the sphere the enthalpy (temperature) of the surface coincides with the enthalpy of gas and is equal to the value  $h_w$  (there is no jump in temperature) and both velocity components are equal to zero

(there is no slip). On the axis  $\theta=0$ ,  $\theta=\pi$  we use the conditions of symmetry of the flow. If  $\theta^* < \pi$ , it is necessary to assign conditions for the functions  $u$ ,  $v$ ,  $h$  along the ray  $\theta=\theta^*$ . By means of series of calculations with a varied position of the angle  $\theta^*$ ,  $0 < \theta^* < \pi$  it was established that, when assigning the approximate conditions for these functions in the form of  $\partial^2 f / \partial \theta^2 = 0$  (i. e., a linear change in  $f$  in the vicinity of the ray  $\theta^*$ ) there is an insignificant reverse effect of the flow. In this case the density  $\rho(r, \theta^*)$  can be found from an analogous condition or from a continuity equation.

The finite-difference approximation of the initial equations and boundary conditions, and also the peculiarities of the numerical algorithm, are described in [1]–[3].

2. The study of the super- and hypersonic flow around blunt solids with the cooling of their surface ( $h_w < h_0$ ) is of practical interest in connection with the problem of heat protection for a flying vehicle. Below we'll discuss the characteristic peculiarities of the viscous gas flow near the sphere placed in a supersonic flow of a diatomic ideal gas ( $\gamma=1.4$ ,  $Pr=0.72$ ,  $\omega=3/4$ ,  $h_w=0.3$ ). The effect of the  $M_\infty$  and  $Re_\infty$  numbers has been studied on the flow pattern under the conditions of flow and heat exchange, which are indicated in the table. The same table shows the Knudsen  $Kn$  and Reynolds  $Re_s$  (calculated using the parameters behind the normal shock) numbers which characterize the degree of rarefaction of the incident flow:

$$Re_s = \frac{\rho_s u_{s'} r_w}{\mu_s} = \frac{Re_\infty}{\bar{\mu}_s} = \frac{Re_\infty}{M_\infty^2} \left[ \frac{\bar{\rho}_s (\gamma+1)}{2\gamma - (\gamma-1)/M_\infty^2} \right]^{\omega}.$$

$$\bar{\rho}_s = \frac{\rho_s}{\rho_\infty}, \quad \bar{\mu}_s = \frac{\mu_s}{\mu_\infty}, \quad Kn = \frac{l_\infty}{r_w} = \frac{16}{5} \sqrt{\frac{\gamma}{2\pi}} \cdot \frac{M_\infty}{Re_\infty}.$$

With an increase in rarefaction there is an increase in the  $Kn$  number and a decrease in  $Re_s$ . In most calculations  $\theta^*=99^\circ$ .

Let's briefly describe the behavior of the gas-dynamic parameters at the leading critical point in the area where the shock wave is thickened and in a compressed layer behind it, and also along the surface of the sphere. Since the calculations were accomplished for the flow conditions which occupy the intermediate position



between the flows of the solid and free-molecular media, it is of interest to compare the results of the calculation with the corresponding theoretical results and experiments in rarefied gases.

1. **Leading critical point.** The solid line in Fig. 2, a represents the behavior of the ratio  $p_w^0/p'_0$ , which characterizes the deviation of the calculated pressure from the pressure in an inviscid flow, as a function of  $Re_s \sqrt{V \rho_s}$ . With a decrease in the parameter  $\eta = Re_s \sqrt{V \rho_s}$  (introduced for a convenience of the comparison with the experimental data [5], shaded area in this figure) the values of  $p_w^0/p'_0$  begin to increase approximately from  $\eta=100$ , which is in good agreement with the experiments and the theory of free-molecular flows [6] (with  $Re_s=0$ ,  $\gamma=1.4$  and totally diffused deflection of the molecules from a hard, very cold wall the ratio  $p_{CM}/p'_0$  attains a value equalling to 1.087). An increase in pressure is the result of merging of the thickened boundary layer and shock wave.

The solid line in Fig. 2, b shows the behavior of the ratio  $q_w^0/q_{nc}^0$  as a function of the  $Re_s$  number. This ratio characterizes the difference in heat transfer in a flow of rarefied gas and in flows with large Reynolds numbers. The value  $q_{nc}^0$  was calculated by the Fay and Riddell formula [7], and

$$q_w(\theta) = \frac{1}{Pr Re_w} \left( \mu \frac{\partial h}{\partial r} \right)_w, \quad 0 \leq \theta \leq \theta^*.$$

According to the theory of free-molecular flow  $q_{CM}^0=1/2$  and the ratio  $q_{CM}^0/q_{nc}^0$ , which is proportional to  $\sqrt{Re_s}$ , approaches zero with an increase in rarefaction. The same figure shows the following: the shaded area - data of a series of experiments (borrowed from [5], the dot-dash line -  $q_{CM}^0/q_{nc}^0$ , the dashed line - the curve

$q_w^0/q_{nc}^0 = 1 + 0.52/\sqrt{Re_s}$ , obtained in [8] by processing the measurements (the conditions of this experiment are close to the conditions of these calculations:  $M_\infty=2, 4, 6$ ;  $20 \leq Re_s \leq 800$ ,  $T_w/T_\infty \approx 0.33$ ). In most of the experiments carried out to determine heat transfer the value  $q_w^0/q_{nc}^0$  has the following nature of change: with a decrease in  $Re_s$  ( $0 \leq Re_s \leq 10^3$ ) this value increases, here  $q_w^0 > q_{nc}^0$ , attains

a maximum at certain  $\overline{Re}_s$  ( $10 \leq \overline{Re}_s \leq 10^2$ ), and then decreases ( $q_w^0 < q_{nc}^0$  when  $Re_s \leq 10$ ). The calculated values of  $q_w^0 / q_{nc}^0$  detect a similar behavior; although they begin to decrease when  $Re_s \leq 10$ , they still remain higher than those determined in the experiments (with the same  $Re_s$  number). Apparently, at low Reynolds numbers,  $Re_s < 10^2$ , one should take into account in the calculation the conditions of slip and temperature jump on the wall, which, in particular, lead to a decrease in the values of the heat flow [9].

With a decrease in the  $Re_s$  number the density  $\rho_w^0$  increases in proportion to  $p_w^0$  since the  $h_w$  is constant (with thermal insulation a drop was observed in the density at the leading critical point [4] and [10]).

2. Shock wave structure. The thickness of a shock wave in a viscous gas is usually determined by the formula proposed by Prandl:

$$d = \frac{f_s - f_\infty}{(\partial f / \partial r)_{\max}}.$$

where  $f$  - any of the gas-dynamic functions,  $\partial f / \partial r$  - maximum slope of the profile  $f$  to the axis  $\theta=0$ . In most of the experiments carried out to study the shock-wave structure (for example, [11] and [12]) the density field was measured and, therefore, thickness  $d$  was calculated using the profile  $\rho$  in the area of the shock wave. However, under the cooling conditions of the surface the calculated density profiles  $\rho(r, 0)$  in the shock wave have a complex behavior (see Figs. 4 and 5) and it is difficult to use them for the calculation of  $d$ . For this it is more convenient to use the profiles  $u(r, 0)$  and  $p(r, 0)$ , which have approximately the same nature as with thermal insulation. The calculation of the thickness of the shock wave using the profiles  $u(r, 0)$ , when  $M_\infty=6$  and  $Re_\infty=300, 200, 90, 45$ , and  $20$ , yield the following values:  $d=0.055; 0.084; 0.169; 0.286$ ; and  $0.474$ . Thus, with an increase in rarefaction the shock wave thickens quickly. The comparison of the thickness  $d$  in cases of thermal insulation and cooling (with all other conditions being equal) indicates a somewhat lesser thickness in the case of cooling. The thickness of the transition zone in the shock wave, when there is a change in the angle  $\theta$ , can be determined conditionally by the

value  $D(\theta) = r_1(\theta) - r_2(\theta)$ , where  $r_1$  and  $r_2$  are points at the fixed ray  $\theta$ : in  $r_1$  the value  $p - p_\infty \approx 0.01 p_\infty$ , in  $r_2$  the  $\partial p / \partial r$  attains a maximum value in the shock wave. The position of the washed out shock wave relative to the sphere can then be determined by the middle line  $r_D$  of the indicated zone. The dot-dash curve on Fig. 6 shows the position of the line  $r_D$  when  $Re_\infty = 45$  and  $M_\infty = 6$ . This line travels in approximately the same way as a shock wave during an inviscid flow but it is located further from the sphere. A similar behavior of the shock wave zone was observed in an experiment (see [5]). With a change in the numbers  $M_\infty$  and  $Re_\infty$  the behavior of the value  $\Delta = r_D - 1$ , which can be roughly considered as the "departure" of the shock wave, has the following feature: a) with a decrease in the number  $Re_\infty$  ( $M_\infty, \theta$  are constant)  $\Delta$  increases; b) with an increase in  $\theta$  ( $Re_\infty, M_\infty$  are constant)  $\Delta$  increases; c) with a change in the number  $M_\infty$  ( $\theta, Re_\infty$  are constant) this function is not monotonic (see Fig. 4): for  $2 \leq M_\infty \leq 4$  the  $\Delta$  diminishes, for  $M_\infty > 4$  it increases (the same behavior was observed in the calculations of a thermoinsulated sphere [4]). The shock wave intensity was determined by the value of the ratio  $I(\theta) = (p_{\max} - p_\infty) / (p'_0 - p_\infty)$ , where  $p_{\max}$  is the value of maximum pressure in the area of the shock wave at the fixed ray  $\theta$ . It was found that with an increase in  $\theta$  the function  $I(\theta)$ , for all the flow conditions examined, decreases (for example, when  $M_\infty = 6$ ;  $I(0) = 0.917$ ;  $I(\frac{\pi}{2}) = 0.238$  for  $Re_\infty = 200$  and  $I(0) = 0.882$ ,  $I(\frac{\pi}{2}) = 0.193$  for  $Re_\infty = 45$ ).

Thus, with an increase in the angle  $\theta$  and a decrease in the  $Re_\infty$  number, there is a continuous (according to  $r$ ) washing out and weakening of the shock wave (just as in the case of thermal insulation [1]–[3]).

3. Structure of a compressed layer. Figure 3 (a, b, c) shows a typical behavior of the flow parameters in a compressed layer ( $M_\infty = 8$ ,  $Re_\infty = 90$ ), where, for comparison, the dashed lines show the solution when  $\theta = 0$  in the case of thermal insulation of the sphere. From an analysis of the calculated data we draw the following conclusions. The enthalpy  $h(r, \theta)$  has a maximum on each ray  $\theta$  due to



the cooling of the sphere's surface. The density  $\rho(r, \theta)$  has a local maximum immediately behind the shock wave as it moves away from the region of the leading critical point, and maximum pressure  $p(r, \theta)$  is attained between the shock wave and wall; with the low values of  $\theta$  the maximum pressure  $p(r, \theta)$  is on the wall. With a relatively large  $Re_s$  numbers (for example, when  $M_\infty=2$  and  $Re_\infty=500$ ) there is an area close to the sphere's surface in which the pressure along the normal to the wall changes insignificantly, which indicated the existence of a region of the type of a boundary layer.

The position of the sonic line  $r_*(\theta)$  is depicted in Fig. 6 for the various  $Re_\infty$  ( $M_\infty=6$ , solid lines) numbers and in Fig. 8 for the various  $M_\infty$  ( $Re_\infty=90$ , solid lines) numbers. It is evident that with a change in  $\theta$  and  $M_\infty$  ( $Re_\infty$  is constant) the behavior of these lines is nonmonotonic. For a constant  $M_\infty$  number sonic lines move further away from the sphere's surface with a decrease in the  $Re_\infty$  number.

Let us examine in more detail the behavior of the solution along the stagnation line  $\theta=0$ . Figure 4 characterizes the behavior of  $u$  and  $\rho$  with a change in the  $M_\infty$  ( $Re_\infty=90$ ) number. Using profiles  $u(r, 0)$  we can clearly see the nonmonotonic nature of "departure" of the shock wave noted above. The density increases along the  $\theta=0$  axis when  $r \rightarrow 1$ ; here there are regions of sharp increase of  $\rho$  in the shock wave and close to the body (the latter is the result of cooling of the surface). Both these regions have a tendency to merge with an increase in the  $M_\infty$  number and the fixed  $Re_\infty$  number (i. e., with a decrease in the  $Re_s$  number) (Figs. 4 and 5).

Figure 5 shows the distribution of the density  $\rho(r, 0)$  and enthalpy  $h(r, 0)$  when  $M_\infty=6$  and a change in the  $Re_\infty$  number. It is evident that with a decrease of the  $Re_\infty$  number (and, consequently,  $Re_s$  number) the maximum value of  $h(r, 0)$ , which is achieved behind the shock wave, also decreases and  $h_{\max} < h_s$ . If we take the point  $r$ , where the maximum  $h(r, 0)$  is achieved, to be the trailing front of the washed-out shock wave, it turns out that, at this point, the density  $\rho(r, 0) < \rho_s$  and  $p(r, 0) < p_s$ , i. e., an incomplete compression in the shock wave, leads to disruption of the Rankin-Hugoniot conditions and to its gradual disintegration with an increase in rarefaction. With an increase in rarefaction its gradual thicken-

ing is evident in the area of the shock wave according to the profiles of  $h(r, 0)$ . Approximately the same behavior of the density and temperature was observed along the line of stagnation in the experiment of work [13], in which nitrogen flowed past a cylinder with a spherical blunt end ( $M_\infty=9$ ,  $Kn=0.045$ ,  $T_0=298^\circ K$ , and  $T_w/T_0 \approx 0.3$ ). The field temperatures determined in [3] attest to the presence of a noticeable temperature jump on the model surface ( $T_w^0/T_0=0.78$  when  $T_w/T_0 \approx 0.3$  at the leading critical point).

4. Distributions along the surface of the sphere. Figure 7 shows the graphs of the deduced pressure  $\bar{p}_w = p_w(\theta)/p_w(0)$  and heat flow  $q_w(\theta)$  when  $M_\infty=6$  and with different values of the  $Re_\infty$ . In the same figure the dot-dash line depicts the function  $q_{CM} = \cos\theta/2$ , the dotted line -  $q_{nc}(\theta)$ , obtained from the calculation of the boundary layer on the sphere [14] ( $M_\infty=10$ ,  $Re \sim 10^6$ ,  $\gamma=1.4$ ,  $T_w/T_0=0.6$ ), the dashed line -  $\bar{p}_w(\theta)$  when  $M_\infty=6$ ,  $Re_\infty=\infty$  ([7], chapter 8). With a decrease in the  $Re_\infty$  number the values of  $\bar{p}_w$  gradually increase on the front section of the sphere and they are higher than the corresponding values in an inviscid flow. With an increase in the angle  $\theta$  the function  $q_w(\theta)$  decreases gradually and with a decrease in the  $Re_\infty$  the values of  $q_w(\theta)$  increase; in this case,  $q_{nc} < q_w < q_{CM}$ . The nature of change in the calculated heat flow is in agreement with the experiments [8] and [15].

Figure 8 shows the friction coefficient graphs  $C_f = \tau_w / \rho_\infty W_\infty^2 = Re_\infty^{-1} (\mu \partial v / \partial r)_w$  for  $Re_\infty=90$  and the various  $M_\infty$  numbers (solid lines). In the same figure the dot-dash line depicts the function  $[C_f(\theta)]_{CM} = (\sin 2\theta)/2$ , the dashed line -  $[C_f(\theta)]_{nc}$  taken from work [14]. All calculation curves of  $C_f(\theta)$  are located between the curves for the free-molecular and boundary-layer conditions of flow. The values of  $C_f$  increase with a decrease in the  $Re_\infty$  number. With a change in the angle  $\theta$  the function  $C_f(\theta)$  has a maximum for the various  $Re_\infty$  numbers when  $\theta \sim 40-60$  degrees, i. e., about in the middle of the area of a sharp approach of the sonic line to the sphere. In the calculations of the boundary layer on blunted solids [16] it was found that the maximum values of the function  $C_f(\theta)$  were obtained in the vicinity of the sonic point on the surface



of the streamlined body. The behavior of the functions  $\bar{p}_w$  and  $C_f$  is analogous to the case of thermal insulation when there is a decrease in the  $Re_\infty$  number [4].

Figure 9 shows the construction of the ratio  $C_D/(C_D)_{CM}$  as a function of the number  $2Re_s$ , the shaded areas indicate the data of the experiments carried out to determine the resistance of spheres [5]. The quantity  $C_D$  was calculated by the formula  $C_D = 4 \int_0^{\theta^*} (p_w \cos \theta + C_f \sin \theta) \sin \theta d\theta$ , and  $(C_D)_{CM}$  by the formula used to calculate the diffused deflection with the accommodation coefficient equalling unity [6]. It is obvious that in the examined conditions of flow  $C_D < (C_D)_{CM}$ , while the ratio being examined increases with a decrease in the  $Re_s$  number.

3. The numerical results indicate that when  $Re_s \leq 10^2$  the shock wave and the boundary layer join in the area of the leading critical point of the sphere, which leads to a series of new phenomena which do not take place in flows with large Reynolds numbers. With reduced rarefaction the calculated aerodynamic characteristics have a tendency to approach the corresponding values in a free-molecular flow and, in the range  $10 \leq Re_s \leq 10^2$ , there is a satisfactory agreement between them and the measurements derived from the experiments. When  $Re_s \leq 10$  there is a certain anomaly in the behavior of some characteristics of the flow, which can be explained by the fact that the effects of slip and temperature jump were disregarded in these calculations. Apparently, when taking these effects into consideration within the framework of the Navier-Stokes model, there should be a better agreement with the experiments also at lower Reynolds numbers ( $Re_s < 10$ ). The same conclusions are drawn from the analysis of flows with thermal insulation of the sphere's surface [4]. Thus, one can assert that the numerical solutions of the problem of flow in this setting afford the description of the phenomena in a compressed layer, which are close to those observed in experiments.

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results.

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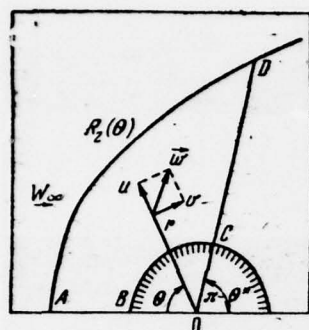


Fig. 1.

Table

$M_{\infty}$	$Re_{\infty}$	$h_w/h_0$	$Re_s$	$Kn$
2	500	0,267	338	0,006
2	90	0,267	61	0,033
4	300	0,457	105	0,020
4	90	0,457	32	0,067
6	300	0,527	63	0,030
6	200	0,527	42	0,045
6	90	0,527	19	0,100
6	45	0,527	9,5	0,200
6	20	0,527	4,2	0,450
8	90	0,557	13	0,133

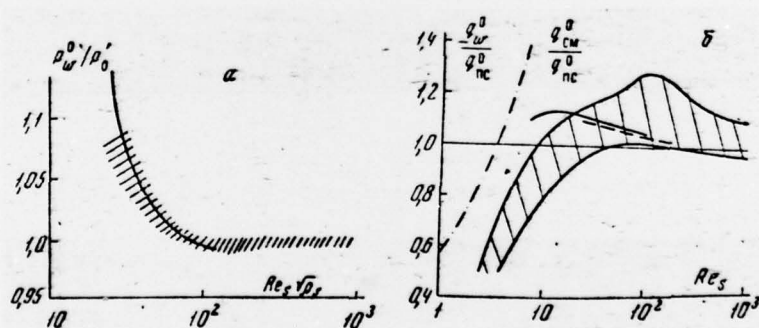


Fig. 2. Behavior of  $\rho_w^0/\rho_0^0$  (a) and  $q_w^0/q_{nc}^0$  (b) at the leading critical point.

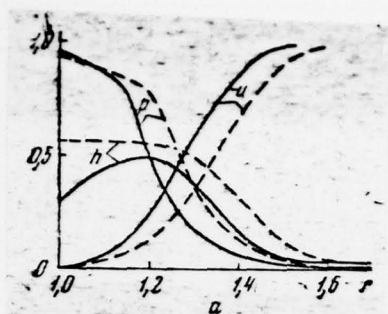
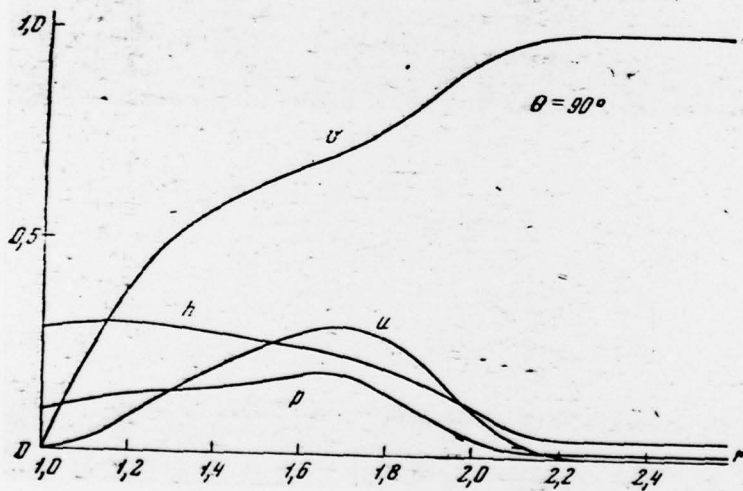
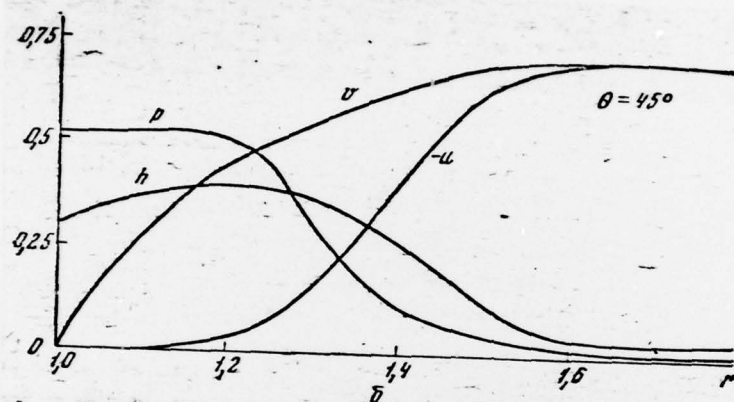


Fig. 3. Parameters of flow in a compressed layer.  $M_\infty=8$ ,  $Re_\infty=90$   
(a -  $\theta=0^\circ$ , b -  $\theta=45^\circ$ , c -  $\theta=90^\circ$ )





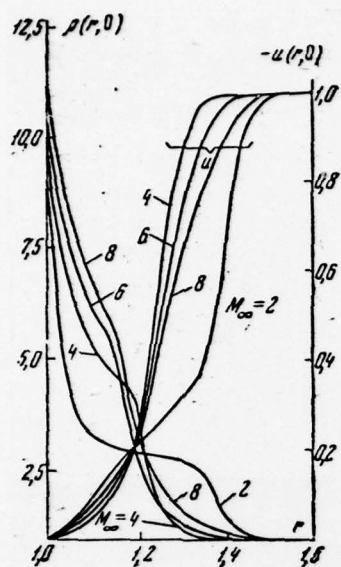


Fig. 4. Profiles of functions  $\rho$  and  $u$  on the stagnation line  $\theta=0$  ( $Re_\infty=90$ )

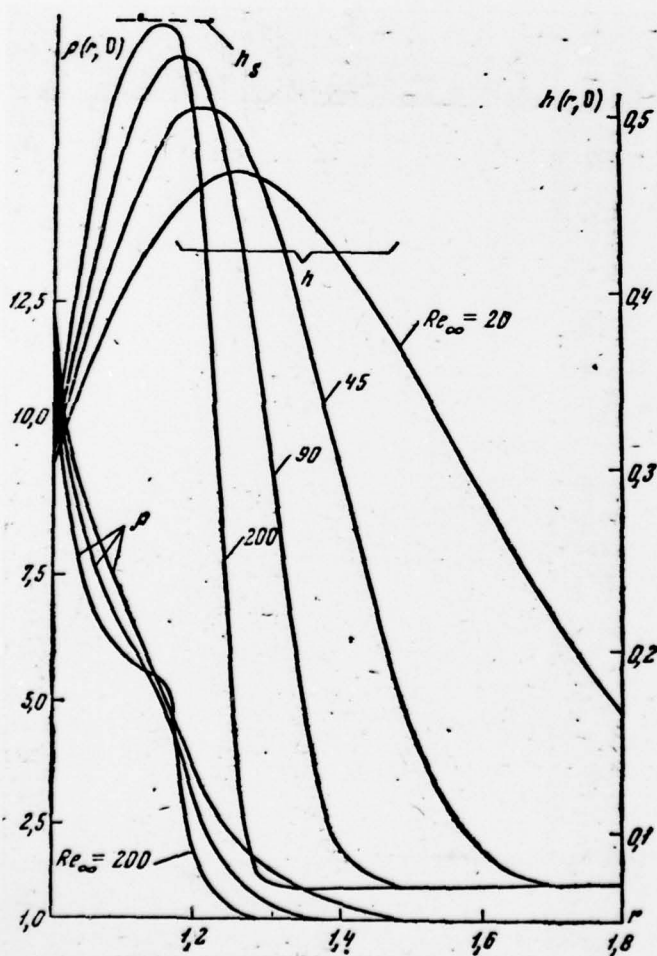


Fig. 5. Profiles of functions  $\rho$  and  $h$  on the stagnation line  $\theta=0$  ( $M_\infty=6$ )



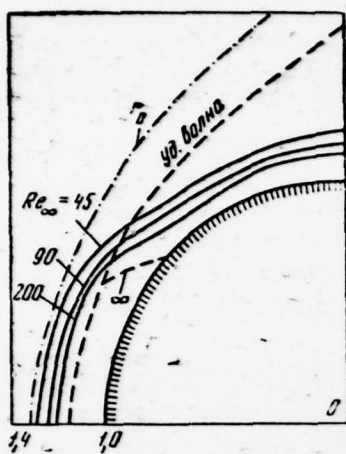


Fig. 6. Sonic lines,  $M_\infty=6$

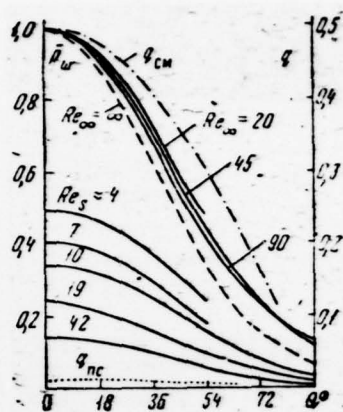


Fig. 7. Distribution of pressure  $\bar{p}_w$  and heat flow  $q_w$  on the sphere

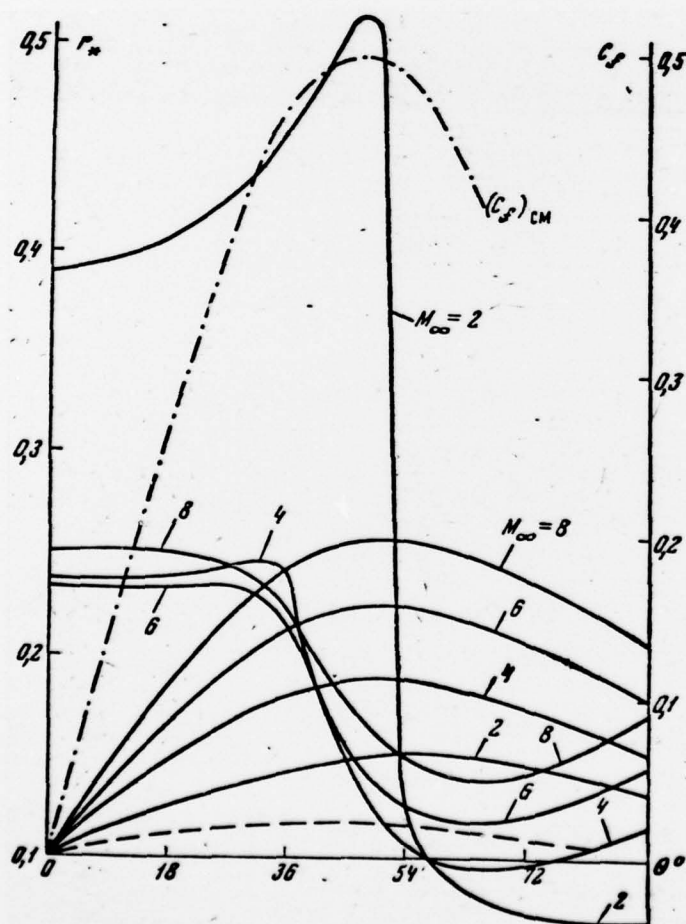


Fig. 8. Sonic lines  $r_*$  and friction coefficient  $C_f$  ( $Re_\infty=90$ )

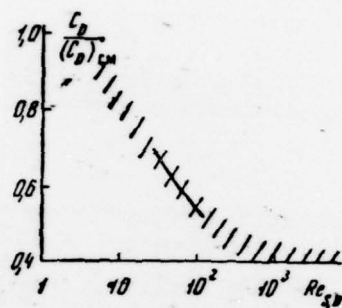


Fig. 9. Sphere resistance coefficient

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